Аксиоматический и эмпирический анализ способов многокритериального ранжирования в контексте проблемы оценки научных журналов

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(a)   How to construct a reasonably good representation of the set of rankings which are based on bibliometric indicators?

(b)   What happens to the aggregates if the Hirsch index is excluded from the set of aggregated bibliometric indicators?

Indicator	Database	Year	Publication window, years	Weighted	Size-dependent
impact factor	WoS/JCR	2011	2	No	No
5-year impact factor	WoS/JCR	2011	5	No	No
immediacy index	WoS/JCR	2011	1	No	No
article influence	WoS/JCR	2011	5	Yes	No
Hirsch index	WoS	2007–2011 (papers and citations)	5	No	Yes
SNIP	Scopus	2011	3	No	No
SJR	Scopus	2011	3	Yes	No

- Economics: 212 journals
- Management: 93
- Political Science: 99

Share of inversions, % (economic journals)

	impact factor	5-year impact factor	immediacy index	article influence	Hirsch index	SNIP	SJR
impact factor		**8.46**	24.59	18.13	15.45	15.09	14.23
5-year impact factor	**8.46**		24.25	13.72	13.15	13.66	12.20
immediacy index	24.59	24.25		26.00	25.57	**27.01**	25.25
article influence	18.13	13.72	26.00		17.15	16.31	15.50
Hirsch index	15.45	13.15	25.57	17.15		18.47	15.05
SNIP	15.09	13.66	**27.01**	16.31	18.47		17.28
SJR	14.23	12.20	25.25	15.50	15.05	17.28	

X – the *general set* of alternatives

A – the *feasible set* of alternatives: $A \subseteq X \wedge A \neq \varnothing.$ The feasible set is a variable.

N – the *society* (a group of voters or a panel of experts)

$u_i(x)$ – the *utility* of alternative $x \in X$ for voter $i \in N$, $u_i(x): X \rightarrow \mathbb{R}$

$$u_i(y) > u_i(x) \Leftrightarrow \text{voter } i \text{ strictly prefers } y \text{ to } x$$

$U = \{\, u_i(x) \mid i \in N \,\}$ – the profile of utility functions

R – (*weak*) *social preferences*, $R \subseteq X \times X$

R is presumed to be complete: $\forall\, x \in X,\ \forall\, y \in X,\ (x, y) \in R \vee (y, x) \in R$

P – *strict social preferences*, $P \subseteq R$: $(x, y) \in P \Leftrightarrow ((x, y) \in R \wedge (y, x) \notin R)$

It is presumed that $R = R(P)$ and $P = P(U)$.

Aggregation rule R=R(U)

- **Completeness**: all alternatives are comparable, $xR(U)y \lor yR(U)x$

- **Transitivity**: $(xR(U)y \land yR(U)z) \Rightarrow xR(U)z$

- **Neutrality**: the rule treats all alternatives equally

- **Anonymity**: the rule treats all aggregated rankings equally

- **Strong Pareto principle**: if x Pareto-dominates y, then xPy

- **Full domain**: the rule can be applied in all cases, i.e. to any utility profile U

- **Independence of irrelevant utilities**: $\forall A \subseteq X$, $P(U)|_A = P(U|_A)$

- **Ordinality**: if utility profiles U and U' are such that $\forall x, y \in A \subseteq X$, $\forall i \in N$, $u_i(x) > u_i(y) \Leftrightarrow u'_i(x) > u'_i(y)$, then $R(U|_A) = R(U'|_A)$ for any such $A \subseteq X$.

N – the set of indicators; $u_k(x)$ – the value of indicator *k* for journal *x*
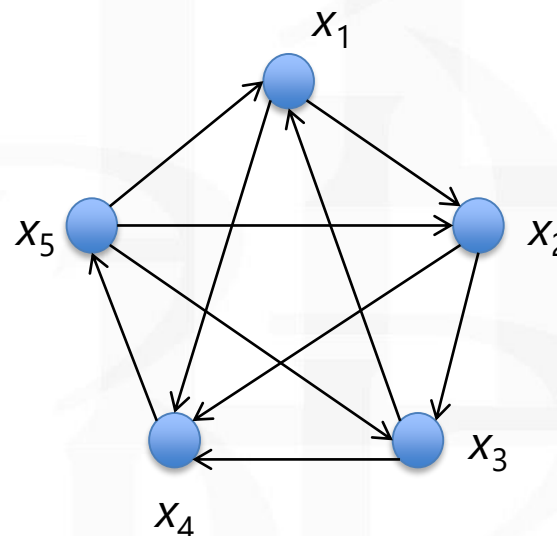
The majority rule

x is better than $y \Leftrightarrow \# \{ k \in N \mid u_k(x) > u_k(y) \} > \# \{ k \in N \mid u_k(y) > u_k(x) \}$

P – **the majority relation**: $(x, y) \in P \Leftrightarrow x$ is majority-preferred to *y*

M=$[m_{ij}]$ – matrix representing *P*: $m_{xy}=1 \Leftrightarrow (x, y) \in P$, $m_{xy}=0 \Leftrightarrow (x, y) \notin P$

	x_1	x_2	x_3	x_4	x_5
x_1	0	1	0	1	0
x_2	0	0	1	1	0
x_3	1	0	0	1	0
x_4	0	0	0	0	1
x_5	1	1	1	0	0

Tournament matrix **M**

Majority digraph

- *Completeness*

- ~~*Transitivity*~~

- *Neutrality*

- *Anonymity*

- *Strong Pareto principle*

- *Full domain*

- *Independence of irrelevant utilities*

- *Ordinality*

- *Strict Cardinal Monotonicity*

- *Positive responsiveness*

- *Computational Simplicity*

- **_Strict Cardinal Monotonicity_**: if utility profiles U and U' are such that

  $\forall i \in N, u'_i(x) \geq u_i(x) \wedge u'_i(y) = u_i(y),$

  then $xP(U)y \Rightarrow xP(U')y$ and $xR(U)y \Rightarrow xR(U')y$

- **_Positive responsiveness_**: if utility profiles U and U' are such that

  $\exists j \in N: (u_j(x) < u_i(y) \wedge u'_j(x) \geq u'_j(y)) \vee (u_j(x) = u_j(y) \wedge u'_j(x) > u'_j(y))$ and

  $\forall i \in N \setminus \{j\}, u'_i(x) = u_i(x) \wedge u'_i(y) = u_i(y)$ and $xR(U)y$ and $yR(U)x$ then $xP(U')y$

- **_Computational simplicity_**: there exists a polynomial algorithm

  for computing $R(U)$.

The majority rule (example)

№	Journal	IF	5-IF	Immediacy index	Article influence	Hirsch	SNIP	SJR
1	Explorations in Economic History	**0.935**	0.898	**0.541**	0.772	7	**1.768**	**0.036**
2	Review of Income and Wealth	0.805	**1.103**	0.205	**0.850**	**9**	1.712	0.034

4 > 3

J_1 is better than J_2

The Condorcet paradox

Journal	IF	5-IF	Immediacy index	Article influence	Hirsch	SNIP	SJR
Explorations in Economic History	**0.935**	0.898	**0.541**	0.772	7	**1.768**	**0.036**
Review of Income and Wealth	0.805	1.103	0.205	0.850	9	1.712	0.034
Scandinavian Journal of Economics	0.514	**1.070**	0.150	**1.310**	8	1.426	**0.043**

J_1 is better than J_2 $(4 > 3)$

J_2 is better than J_3 $(5 > 2)$

J_3 is better than J_1 $(4 > 3)$

All seven rankings are aggregated

	3-step cycles	4-step cycles	5-step cycles	Tied pairs	All pairs
Economics	2446	22427	226103	197	22366
Management	203	787	3254	33	4278
Political Science	149	430	1344	73	4851

Six rankings are aggregated (without *h*-index)

	3-step cycles	4-step cycles	5-step cycles	Tied pairs	All pairs
Economics	167	822	3140	1248	22366
Management	19	36	57	212	4278
Political Science	21	58	142	261	4851

- The ***Copeland rule***
  (ranking by the number of victories won in a tournament P)
    *Version **2*** (a tie is counted as a loss)
    *Version **3*** (a tie is counted as a victory)

- A sorting based on a ***tournament solution***,
  which determines the winners of a tournament P
  The best alternatives (the "winners") are determined by
    the ***uncovered set*** UC
    the ***minimal externally stable*** set MES

- Ranking the nodes of a digraph representing P by
  Markovian random walk method

- *Completeness*

- *Transitivity*

- *Neutrality*

- *Anonymity*

- *Strong Pareto principle*

- *Full domain*

- ~~*Independence of irrelevant utilities*~~

- *Ordinality*

- *Strict Cardinal Monotonicity*

- ~~*Positive responsiveness*~~

- *Computational Simplicity*

- *Weak Arrowian Independence irrelevant alternatives*

- ## *Arrowian Independence of irrelevant alternatives*

  AIIA $\Leftrightarrow$ Independence of irrelevant utilities $\wedge$ Ordinality

  $$\forall A \subseteq X, \; \forall x, y \in A, \; \forall i \in N, \; xR_iy \Leftrightarrow xR'_iy \; \wedge \; xP_iy \Leftrightarrow xP'_iy$$
  $$\Rightarrow$$
  $$xR(U|_A)y \Leftrightarrow xR(U'|_A)y \; \wedge \; xP(U|_A)y \Leftrightarrow xP(U'|_A)y.$$

- ## *Weak Arrowian Independence of irrelevant alternatives*

  Suppose the feasible set A is fixed. Then $\forall x, y \in A,$

  $$\forall i \in N, \; \forall z \in A, \; xR_iz \Leftrightarrow xR'_iz \wedge xP_iz \Leftrightarrow xP'_iz \wedge yR_iz \Leftrightarrow yR'_iz \wedge yP_iz \Leftrightarrow yP'_iz$$
  $$\Rightarrow$$
  $$xR(U|_A)y \Leftrightarrow xR(U'|_A)y \; \wedge \; xP(U|_A)y \Leftrightarrow xP(U'|_A)y.$$

- ***Completeness***

- ***Transitivity***

- ***Neutrality***

- ***Anonymity***

- ~~***Strong Pareto principle***~~

- ***Full domain***

- ~~***Independence of irrelevant utilities***~~

- ***Ordinality***

- ~~***Strict Cardinal Monotonicity***~~

- ~~***Positive responsiveness***~~

- ***Computational Simplicity***

- ***Weak Pareto principle***

   if x Pareto-dominates y, then xRy

- ***Independence of classes of irrelevant alternatives***

- ***Cardinal Monotonicity***: if profiles U, U' are s.t. $\forall i \in N,\ u'_i(x) \geq u_i(x) \wedge u'_i(y) = u_i(y)$, then $xR(U)y \Rightarrow xR(U')y$

- **Idempotency**: $\forall A$, $S(S(A))=S(A)$.

- **The Aizerman-Aleskerov condition**: $\forall A$, $\forall B$, $S(A)\subseteq B\subseteq A \Rightarrow S(B)\subseteq S(A)$.

- **Nash Independence of irrelevant alternatives** (*I. of outcasts*): $\forall A$, $\forall B$, $S(A)\subseteq B\subseteq A \Rightarrow S(B)=S(A)$.

NIIA $\Longleftrightarrow$ Idempotency $\wedge$ the Aizerman-Aleskerov condition

If a ranking rule *R* is a sorting based on a tournament solution *S* then

R satisfies *Independence of classes of irrelevant alternatives* and

(Cardinal/Ordinal) *Monotonicity* if *S* satisfies the *Nash IIA*.

MES satisfies the *Nash IIA*.

- *Completeness*

- *Transitivity*

- *Neutrality*

- *Anonymity*

- *Strong Pareto principle*

- *Full domain*

- ~~*Independence of irrelevant utilities*~~

- *Ordinality*

- ~~*Monotonicity*~~

- ~~*Positive responsiveness*~~

- *Computational Simplicity*

Kendall τ_b (economic journals)

	impact factor	5-year impact factor	immediacy index	article influence	Hirsch index	SNIP	SJR	Copeland (2)	Copeland (3)	UC	MES	Marcovian
impact factor	1.000	0.830	0.503	0.637	0.654	0.698	0.700	0.834	0.831	0.834	0.835	0.819
5-year IF	0.830	1.000	0.510	0.725	0.702	0.726	0.741	0.903	0.904	0.906	0.896	0.891
immediacy index	0.503	0.510	1.000	0.475	0.442	0.454	0.472	0.550	0.551	0.556	0.578	0.560
article influence	0.637	0.725	0.475	1.000	0.620	0.673	0.674	0.766	0.769	0.777	0.785	0.769
Hirsch index	0.654	0.702	0.442	0.620	1.000	0.592	0.650	0.738	0.737	0.737	0.747	0.729
SNIP	0.698	0.726	0.454	0.673	0.592	1.000	0.638	0.759	0.759	0.767	0.775	0.750
SJR	0.700	0.741	0.472	0.674	0.650	0.638	1.000	0.792	0.790	0.800	0.797	0.775
Copeland (2)	0.834	0.903	0.550	0.766	0.738	0.759	0.792	1.000	0.990	0.970	0.950	0.956
Copeland (3)	0.831	0.904	0.551	0.769	0.737	0.759	0.790	0.990	1.000	0.969	0.950	0.959

Kendall τ_b (economic journals)

	IF	5-IF	Immediacy	AI	Hirsch	SNIP	SJR
5-year IF	**0.830**	**1.000**	0.510	0.725	0.702	0.726	0.741
Markovian	0.819	0.891	**0.560**	**0.769**	**0.729**	**0.750**	**0.775**

The Markovian ranking majority-dominates the ranking based on 5-IF

ERGO

The Markovian ranking represents the set of seven single-indicator-based rankings better than the ranking based of 5-year impact factor

rank	Economics	Man. Sc.	Pol. Sc.	Previous results (2008)
1	*MES*	*MES*	*MES*	*UC*
2	*UC*	*UC*	*UC*	*MES*
3	Copeland 3	Copeland 2	Copeland 3	Copeland 3
4	Copeland 2	Copeland 3	Copeland 2	Copeland 2
5	Markovian	Markovian	Markovian	Markovian
6	5-IF	5-IF	5-IF	IF
7	IF	SNIP	Hirsch	5-IF
8	SJR	Hirsch		SJR
9	AI	AI	AI / IF / SJR	
10	SNIP	SJR		AI / Hirsch / SNIP
11	Hirsch	IF	SNIP	
12	Immediacy	Immediacy	Immediacy	Immediacy

rank	Economics	Management	Political Science
1	*UC*	*UC / MES*	*UC*
2	*MES*		*MES*
3	Copeland 2 / Copeland 3	Copeland 3	Copeland 2 / Copeland 3
4		Copeland 2	
5	Markovian	Markovian	Markovian
6	5-IF	5-IF	5-IF
7	IF	SNIP	IF
8	SJR	AI	SJR
9	AI / SNIP	IF / SJR	AI / SNIP
10			
11	Immediacy	Immediacy	Immediacy

The rankings of rankings (Economics)

rank	With *h*-index (**7** aggregated)	Without *h*-index (**6** aggregated)
1	*MES*	*UC*
2	*UC*	*MES*
3	Copeland 3	Copeland 2 / Copeland 3
4	Copeland 2	
5	Markovian	Markovian
6	5-IF	5-IF
7	IF	IF
8	SJR	SJR
9	AI	AI / SNIP
10	SNIP	
11	Immediacy	Immediacy

rank	With *h*-index (**7** aggregated)	Without *h*-index (**6** aggregated)
1	*MES*	*UC / MES*
2	*UC*	
3	Copeland 2	Copeland 3
4	Copeland 3	Copeland 2
5	Markovian	Markovian
6	5-IF	5-IF
7	SNIP	SNIP
8	AI	AI
9	SJR	IF / SJR
10	IF	
11	Immediacy	Immediacy

rank	With *h*-index (**7** aggregated)	Without *h*-index (**6** aggregated)
1	*MES*	*UC*
2	*UC*	*MES*
3	Copeland 3	Copeland 2 / Copeland 3
4	Copeland 2	
5	Markovian	Markovian
6	5-IF	5-IF
7	AI / IF / SJR	IF
8		SJR
9		AI / SNIP
10	SNIP	
11	Immediacy	Immediacy

(a)   The rankings based on popular bibliometric indicator strongly and positively correlate with each other, but there always is a non-negligible percentage of contradictions.

(b)   To construct a good representation of the set of single-indicator-based rankings one may use a majority-rule-based rank aggregation procedure.

(c)   The exclusion of the Hirsch index from the set of indicators does not significantly change the results of such aggregation.

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Спасибо
за внимание!